ActuaRisk
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Performance Attribution

Methodology Overview

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March 2004

Performance Attribution Methodology

1.1 Introduction

Performance Attribution is a set of techniques that performance analysts use to explain why a portfolio's performance differed from the benchmark. This difference between the portfolio return and the benchmark return is known as the active return/excess return/etc. The active return is the component of a portfolio's performance that arises from the fact that the portfolio is actively managed.

For pure equity portfolios, the attribution analysis dissects the value added into three components (or four if there is a currency effect):

- asset allocation captures the pure effect of the portfolio's asset allocation between sectors, without any stock selection effect in the sectors,
- stock selection is the value added by decisions within each sector of the portfolio,
- interaction captures the value added that is not attributable solely to the asset allocation and stock selection decisions.

The three attribution terms (asset allocation, stock selection, and interaction) sum exactly to the active return without the need for any "fudge factors". Obviously, this approach is applicable to bond or mixed portfolios. Nevertheless the models applied for these portfolio types are, in general more complex as investment decisions are different for these portfolios (e.g. duration, quality of issuers, etc).

1.2 GIPS Requirements to Calculate Portfolio Returns

Achieving comparability among investment management firms' performance presentations requires uniformity in methods used to calculate returns. This is the reason why the Global Investment Performance Investment Standards mandate the use of certain calculation methodologies.

Valuing the portfolio each time there is an external cash flow ought to result in the most accurate method to calculate the time-weighted rates of return, referred to as the "true" Time-Weighted Rate of Return Method.

A formula for calculating a true time-weighted portfolio return whenever cash flows occur is:

$$R_{TR} = \left(\frac{EMV - BMV}{BMV}\right), \qquad (2.1)$$

where *EMV* is the market value of the portfolio at the end of the sub-period, before any cash flows in the period, but including accrued income for the period. *BMV* is the market value at the end of the previous sub-period (i.e., the beginning of the current sub-period); including any cash flows at the end of the previous sub-period and including accrued income up to the end of the previous period.

The sub-period returns are then geometrically linked according to the following formula:

$$R_{TR} = \left((1+R_1)\times(1+R_2)\times\ldots\times(1+R_n)\right) - 1, \qquad (2.2)$$

where R_{TR} is the total return and R_1, R_2, ..., R_n are the sub-period returns for sub-period 1 through *n* respectively. Sub-period 1 extends from the first day of the period up to and including the date of the first cash flow. Sub-period 2 begins the next day and extends to the date of the second cash flow, and so forth. The final sub-period extends from the day after the final cash flow through the last day of the period.

This method assumes that the cash flow is not available for investment until the beginning of the next day. Accordingly, when the portfolio is revalued on the date of a cash flow, the cash flow is not reflected in the Ending Market Value, but is added to the Ending Market Value to determine the Beginning Market Value for the next day. If the cash flow is available for investment at the beginning of the day the value of the cash flow should be added to the Beginning Market Value.

Note that some day-weighting methods assume the cash flow is available midday and half weight the cash flow in that day. The GIPS standards do not specify which cash flow recognition method firms must use; however, once a method(s) is chosen and the criteria and assumptions are determined, they must be consistently applied.

Beginning 1 January 2010, this methodology is likely to be required by the GIPS standards. Until 2010, approximation methods are permitted such as the original Dietz method, the Modified Dietz method, the original Internal Rate of Return (IRR) method, and the Modified IRR method.

1.3 Performance Attribution Models

As the concept of performance attribution matures in the investment management industry, standard methodologies are beginning to receive the same scrutiny as the methodologies behind the calculation of total return. While the AIMR has yet to produce a handbook on GIPS compliant attribution, the direction is clear, as client increasingly seek attribution reports whose bottom-line returns match those produced by their performance measurement systems.

Section 1.1 has showed that the calculation of the total return of a portfolio requires the following information: market values of the portfolio and the cash flows occurred during the period. By analogy, to calculate the returns for an asset held in a portfolio, we need to know the market values and cash flows for that asset. Many portfolio managers struggle with the practical problems involved in collecting market values and cash flows at the total portfolio level. Indeed, because these practical problems are so great, GIPS does not anticipate requiring the use of time weighted returns for portfolios until 1 January 2010. However, the magnitude of the data problem becomes much larger in performance attribution. For example, if a portfolio manager wishes to attribute a portfolio by industry, they will require weights and returns for every industry. If the portfolio managers wishe to do stock-level attribution, they will require a consistent set of weights and returns for every stock held in the portfolio. Thus, the task of gathering weights and returns for performance attribution may involve collecting one or two orders of magnitude more data than is required simply to calculate a portfolio return.

As the roles of performance measurement and performance attribution converge, it is important to distinguish between the goals of these two concepts and to determine the best combination in order to preserve the information content that represents the inherent value of each approach.

Attribution is probably fraught with more controversy than just about any other aspect of performance measurement: from geometric vs. arithmetic, to the various linking methods, daily vs. monthly, to security vs. sector level. And one of those areas which hasn't been addressed at length, but is definitely controversial, is holdings-based versus transaction-based attribution.

For some time practitioners have been trying to gain greater insight into the distinctions between these two general approaches to attribution. There appear to be two very different camps:

- the pro-transaction based group, who believe that accuracy can only be achieved by a transaction-based model, and
- the holdings-based group, who feel that (a) the purported accuracy is a myth, (b) that the data requirements are such that you will be introducing error and/or noise into the math, and (c) that any marginal gain in accuracy will be offset by the huge costs.

Defining the holdings-based approach is quite simple: it is an attribution model that relies upon the portfolios beginning-period holdings to derive the attribution effects.

Defining the transaction-based method is a bit more challenging. While we would expect that there is some attempt to capture intra-period activity (and not rely solely on the initial holdings), how we do it and to what extent we capture the transaction details are open to debate.

Some authors suggest that there are various degrees of transaction-based methods and that there is probably a point where the distinction between holdings and transaction methods becomes kind of grey. Indeed, a *daily holdings-based* approach can be labelled as either holdings or transaction-based: *each day the portfolio is revalued, so if stocks are sold/bought, the day after we start with the new portfolio composition*. Obviously, this approach has to be considered as a rather low-level transaction-based model in comparison to an approach which captures 100% of transaction activity: *capture of every transaction (buy/sale, dividend, corporate action, etc), each transaction needs to be labelled for its effect (internal cash flow, external cash flow, income, etc) and must have the right impact date*. This is probably the height of transaction-based attribution. We are capturing all the details.

In the holding-based approach to performance attribution, the portfolio is treated each sub-period on a strictly buy-and-hold basis, and the attribution effects are computed using the standard single-period equations of the Brinson model. These attribution effects are then linked together using a multi-period linking algorithm. Transactions are reflected through the portfolio holdings, which are updated at the end of any day in which there was a transaction. Actual transaction prices, however, are not used to compute returns. This approach is equivalent to using a time-weighted method assuming that all transactions occur at the end of the day at the closing price. While, in principle, these assumptions may lead to discrepancies in total portfolio return if both the daily turnover and daily price volatility are large, in practice, such effects are typically minor.

In the transaction-based approach, sub-periods coincide with the timing of external cash flows. An example of this method would be to assume that external cash flows occur at the start of each day, and to use transaction prices to calculate the daily performance. Since there are no external cash flows during the day, in principle this yields the exact solution to the time-weighted return for the portfolio. Many practitioners assume that this transaction-based approach, by extension, also provides the most accurate solution to performance attribution. However, this assumption is flawed due to one subtle point: although there are no external cash flows in this method, there are invariably internal cash flows as securities are traded throughout the sub-period (i.e. intraday transactions). The sector weights and returns must therefore still be computed using a money-weighted approach, which in turn will generate a source of errors. In other words, although the transaction-based approach will provide the most accurate active return, it will nonetheless contain errors in the components of active return (i.e. attribution effects).

In order to answer to the following question “which approach is better?”, it is important to bear in mind that the objective of performance attribution is to measure the sources of active return as accurately as possible, and not to measure total portfolio return. The key advantage of the holdings-based approach is that it uses time-weighted returns to compute the attribution effects, and hence avoids the errors associated with the use of money-weighted returns. Its disadvantage is that it does not take into account the intraday trading effect. As suggested by Laker (2003), to solve this problem, we can easily add a trading effect in the attribution model, at the global portfolio level. This effect will be equal to the difference between the official return minus the sum of the return calculated within the attribution tool and the fees applied.

1.4 Attribution Guidelines Suggested by David Spaulding

As performance attribution has become an increasingly standard part of a money manager’s performance measurement and reporting function, it’s necessary to have standards to insure that all the relevant details are known and understood by the recipients of the report. Attribution analysis is simply too complex and varied to assume that the recipient will understand how the results were prepared. To accomplish this, David Spaulding (2002/2003) published reasonable standards.

The standards are divided into four main sections: terminology, model selection, disclosures, and supplemental information to a GIPS presentation.

1. *Terminology* – A review of the various terms that are typically used with attribution. An agreement on these has to be obtained to insure uniformity.

2. *Model Selection* – Picking the right attribution model is a critical step in implementing performance attribution analysis. We discuss various model characteristics and calculation issues.

3. *Disclosures* – To comply with these standards, firms must disclose certain information about their attribution model.

4. *Supplemental Information to a GIPS Presentation* – We’ve seen a significant interest in attribution statistics being incorporated into a GIPS presentation, albeit as a recommendation at this stage.

In this document, we present only the *Terminology* and the *Model Selection* parts of the standards.

1.4.1 Terminology

1.A *Performance Attribution* – An analytical process or technique to identify the sources that contribute to a return and/or excess return.

1.B *Relative Performance Attribution* – Attribution of a portfolio relative to a benchmark or index.

1.C *Absolute Performance Attribution* – Attribution of a portfolio alone; also known as "contribution."

1.D *Excess Return* – The difference between a portfolio's return and the return of its benchmark. This value may be calculated either arithmetically (also called "additive") or geometrically (also called "multiplicative"). Excess return is also referred to as "active return" and "alpha."

1.E *Geometric Excess Return* – The difference in return between a portfolio and its benchmark, calculated as follows:

$$ER_G = \frac{1+R_P}{1+R_B} - 1, \tag{2.3}$$

where:
ER_G = Geometric Excess Return,
R_P = Portfolio Return, expressed as a decimal, and
R_B = Benchmark Return, expressed as a decimal.

1.F *Arithmetic Excess Return* – The difference in return between a portfolio and its benchmark, calculated as follows:

$$ER_A = R_P - R_B. \tag{2.4}$$

1.G *Geometric Performance Attribution* – An attribution approach that relies upon the geometric approach to derive excess return. Also referred to as *multiplicative* performance attribution.

1.H *Arithmetic Performance Attribution* – An attribution approach that relies upon the arithmetic approach to calculate excess return. Also referred to as *additive* performance attribution.

1.I *Interaction Effect* – An attribution effect that is used to account for the "interaction" between two or more effects (*e.g.*, between the stock selection effect and asset allocation effect for an equity portfolio). Some models may use this effect for unaccounted-for effects, but this should more properly be referred to as "residual."

1.J *Residual* – There are two ways the term "residue" or "residual" is used. One is for a single period and one for multiple periods. In both cases, it references an unaccounted for amount.

For a single period, it's an unaccounted for amount that may arise because of pricing irregularities between the index and portfolio, effects which are unaccounted for by the model or other factors.

For multiple periods, it's an amount that's unaccounted-for which may arise when linking attribution effects over time. This is typically a problem with arithmetic models but not geometric models.

1.4.2 Model Selection

Because the purpose of attribution analysis is to evaluate the source(s) of a portfolio's return, and because different models can yield different (and sometimes conflicting) results, it is important that the model that is selected conform with the investment approach used for the style of investing for the portfolio being evaluated. Because styles of investing may vary even within a firm, it is not inconceivable that the firm will calculate attribution using different models, depending upon the particular style, asset class, etc. While these standards are not intended to be “calculation standards,” in that they will not dictate that specific models be utilized, there are certain aspects to the calculation which we require.

2.A The attribution model must conform with the investment approach for the portfolio being evaluated.

2.B When calculating absolute performance attribution (also known as contribution), the sum of the contribution values must equal the total return of the portfolio. Mathematically:

$$R_P = \sum_{i=1}^{n} C_i \ . \tag{2.5}$$

where:
C_i = calculated contribution values and
n = number of sectors, securities, etc., being evaluated.

2.C When calculating relative performance attribution using an arithmetic model, the sum of the attribution effects must equal the arithmetic excess return. Mathematically:

$$\sum_{i=1}^{n} AE_i = ER_A \ . \tag{2.6}$$

where:
AE_i = Attribution Effects.

2.D When calculating relative performance attribution using a geometric model, the product of the attribution effects must equal the multiplicative excess return. Mathematically:

$$\prod_{i=1}^{n} AE_i = ER_G \ . \tag{2.7}$$

If a geometric model was employed but an adjustment made so that the effects actually sum to the arithmetic excess return, then this must be stated and the methodology that was employed to accomplish this must be explained.

2.E When linking arithmetic attribution effects over time, the sum of the linked attribution effects must equal the linked arithmetic excess return. Mathematically:

$$\sum_{i=1}^{n} \sum_{t=1}^{m} AE_{i,t} = LR_P - LR_B = LER_A \ . \tag{2.8}$$

where:
AE = Attribution Effects,
i = the Individual attribution effects,
t = Time periods over which effects are being linked,
LR_P = Linked Portfolio Return,
LR_B = Linked Benchmark Return, and
LER_A = Linked Arithmetic Excess Return.

2.F When linking geometric attribution effects over time, the product of the linked attribution effects must equal the linked geometric excess return. Mathematically:

$$\prod_{i=1}^{n} \prod_{t=1}^{m} AE_{i,t} = \frac{1 + LR_P}{1 + LR_B} - 1 = LER_G \ . \tag{2.9}$$

2 References

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