Basel II:
Capital Requirements for Equity Investment Portfolios

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Abstract. The Basel Accords represent landmark financial agreements for the regulation of commercial banks. The main purpose of the accords was to strengthen the soundness and stability of the international banking system by providing a minimum standard for capital requirements. In 2004, the Basel Committee proposed new guidelines, which have become known as *Basel II*. We give a short overview of the Basel II framework and present the different approaches which can be used to determine the amount of regulatory capital needed for equity exposures. These methods vary from simple, rather rule of thumb methods, to more sophisticated and economic-oriented approaches. We compare the regulatory capital consumption of two equity portfolios using the different Basel II-compliant methods. We provide evidence that, as far as regulatory capital consumption for equity exposures is concerned, there is no real incentive for banks to use the more sophisticated and economic-oriented models such as VaR or EVT models.

Keywords: Basel II, Regulatory Capital, Value at Risk, Extreme Value Theory.

1 The Basel II Regulatory Capital Framework

1.1 Overview

In June 2004, Central bank governors and the heads of bank supervisory authorities in the Group of Ten[5] endorsed the publication of the document "International Convergence of Capital Measurement and Capital Standards: a Revised Framework". This publication sets out the details for a new regulatory capital adequacy framework commonly known as Basel II; see Basel Committee on Banking Supervision (2004) or Gordon *et al.* (2004).

The ultimate goal of the Basel II Framework is to promote the adequate capitalisation of banks and to encourage improvements in risk management, thereby strengthening the stability of the financial system. This goal will be accomplished through the introduction of "three pillars" that reinforce each other and that create incentives for banks to enhance the quality of their control processes. The first pillar represents a significant strengthening of the minimum requirements set out in the "1988 Accord", while the second and third pillars represent innovative additions to capital supervision:

- "Pillar 1" is the new regulatory standard for minimal capital requirements. It revises the 1988 Accord's guidelines by aligning the minimum capital requirements more closely to the actual risk of economic loss.
- "Pillar 2" is the supervisory review process. It sets broad principles and some specific rules that force regulators and banks to go beyond the mechanical application of Pilar I. Banks are expected to set-up and document procedures to assess consistently the capital adequacy of their different risky portfolios.
- "Pillar 3" is market discipline. It motivates prudent management by enhancing the degree of transparency in banks' public reporting. These will be required to disclose detailed information on the risks they face and capital adequacy.

Basel II reflects the results of extensive consultations with supervisors and bankers worldwide. It will be the basis for national rule-making and for banking organisations to complete their preparations for the new Framework's implementation.

1.2 The Constituents of Capital

The Basel Committee[6] considers that the key elements of capital on which the main emphasis should be placed are equity capital and disclosed reserves. These are the only elements common to all countries' banking systems; they are wholly visible in the published accounts, they are also the basis on which the market typically assesses the capital adequacy and finally, they have a crucial bearing on profit margins. This emphasis on equity capital and disclosed reserves reflects the importance the Committee attaches to securing a progressive

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[5] The Group of Ten is made up of eleven industrial countries (Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, and the United States) which consult and co-operate on economic, monetary and financial matters.

[6] The Basel Committee on Banking Supervision is a committee of banking supervisory authorities that was established by the central bank governors of the Group of Ten countries in 1975. It consists of senior representatives of bank supervisory authorities and central banks from Belgium, Canada, France, Germany, Italy, Japan, Luxembourg, the Netherlands, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

enhancement in both the quality and the level of the total capital resources that are maintained by major banks.

Notwithstanding this emphasis, the member countries of the Basel Committee also consider that there are several other important and legitimate constituents of a bank's capital base which may be included within the measurement system.
In view of this, the Basel Committee concluded in 1988 that capital, for supervisory purposes, should be defined in two tiers. At least 50% of a bank's capital base must be composed by equity capital and published reserves from post-tax retained earnings (tier 1). The other elements of capital (supplementary capital) will be admitted into tier 2 up to an amount equal to that of the core capital. Each of these elements may be included, or may not be included, by national authorities, at their discretion, and this in the light of their national accounting and supervisory regulations.

These supplementary capital elements[7] are:
a) undisclosed reserves,
b) asset revaluation reserves,
c) general provisions,
d) hybrid (debt/equity) capital instruments,
e) subordinated debt.

In 1996, the Basel Committee introduced a third tier of capital (tier 3) consisting of short-term subordinated debt for the sole purpose of meeting a proportion of the capital requirements for market risks.

1.3 The Risk Weights

The capital requirement ultimately depends on the risk, i.e. a random variable (r.v.), that a bank faces.

The Basel Committee relates capital to the different categories of asset or off-balance-sheet exposures, and in doing so, uses risk weights (*RW*) for these exposures according to broad categories of relative riskiness.

According to Basel II, the regulatory capital CAP_{reg} is determined using the following relation:

$$CAP_{reg} = 8\% \times RW \ . \quad (1)$$

For instance, assume that a bank is facing two risks *X* and *Y*, and that the risk weights corresponding to *X* and *Y* are 250% and 400% respectively. If the exposure relative to the risk *X* is 1000 and the exposure relative to the risk *Y* is 4000, then the capital requirements can be derived directly from (1); see Table 1.

[7] We do not further detail the particular conditions attaching to their inclusion in the capital base.

	Exposure	RW	Required Capital
Risk *X*	1000	250%	200
Risk *Y*	4000	400%	1280
Total	5000	370%	1480

Table 1: This table gives an example of Basel II Capital Requirements.

Table 1 shows that Basel II deals with diversification in a simple way – generally assuming a standard level of diversification and incorporating this into the general calibration of the risk weights. The total capital requirement is obtained by simply adding up the capital required for each individual risk.

In the next section we describe two approaches which can be used by banks to determine the risk weighted assets for equity exposures.

It is clear that inadequate risk weights may put a burden on the further development of the private equity and venture capital industry. However, in this paper, we only discuss to which extent the different models proposed by Basel II differ from each other and we do not assess the adequacy of the proposed risk weights themselves.

2 The Basel II Regulatory Capital Framework for Equity Investment Portfolios

2.1 Two possible approaches

The Basel Committee allows two approaches to calculate risk-weighted assets for the equity investment portfolio (i.e. those equity exposures that do not make up part of the Trading Book): the PD/LGD approach and the market-based approach. Supervisors will decide which approach or approaches will be used by banks, and in what circumstances.

2.2 PD/LGD Approach

The minimum requirements and methodology for the PD/LGD approach for equity exposures are – apart from some specifications – the same as those for the so-called internal ratings-based approach (IRB) for corporate exposures, see Basel Committee on Banking Supervision (2004).

The specifications relative to equity exposures are:

- The bank's estimate of the probability of default (PD) of a corporate entity in which it holds an equity position must satisfy the same requirements as the bank's estimate of the PD of a corporate entity in which the bank holds debt. If a bank does not hold debt of the company in whose equity it has invested, a 1.5 scaling factor will be applied to the risk weights derived from the corporate risk-weight function, given the PD set by the bank. For more details, we refer to Basel Committee on Banking Supervision (2004, p 73).

- A loss given default (LGD) of 90% would be assumed in deriving the risk weight for equity exposures.
- For these purposes, the risk weight is subject to a five-year maturity adjustment (*M*) whether or not the bank is using the explicit approach to maturity elsewhere in its IRB portfolio.

The formula for calculating the risk weights is:

$$RW = \begin{pmatrix} \left(LGD\ \phi(t) - PD\ LGD\right) \\ \times\left(\dfrac{1}{1-1.5b}\right) \\ \times\left(1+(M-2.5)b\right)12.5 \end{pmatrix} + \left(PD\ LGD\right), \qquad (2)$$

where $t = (1-K)^{-\frac{1}{2}}\phi^{-1}(PD) + \left(\dfrac{K}{1-K}\right)^{\frac{1}{2}}\phi^{-1}(0.999)$,

$K = 0.12\dfrac{1-e^{-50\cdot PD}}{1-e^{-50}} + 0.24\left(1-\dfrac{1-e^{-50\cdot PD}}{1-e^{-50}}\right)$,

$b = (0.11852 - 0.05478\ln(PD))^2$,

$M = 5$ and $LGD = 90\%$,

$\phi(x)$ is the distribution function (d.f.) of a standard normal random variable.

A minimum risk weight of 100% applies for public and private equity portfolios as long as the portfolio is managed in the manner outlined below:

- Public equities where the investment is part of a long-term customer relationship, any capital gains are not expected to be realised in the short term and there is no anticipation of (above trend) capital gains in the long term. It is expected that in almost all cases, the institution will have lending and/or general banking relationships with the portfolio company so that the estimated probability of default is readily available. Given their long-term nature, specification of an appropriate holding period for such investments merits careful consideration. In general, it is expected that the bank will hold the equity over the long term (at least five years).
- Private equities where the returns on the investment are based on regular and periodic cash flows not derived from capital gains and there is no expectation of future (above trend) capital gain or of realising any existing gain.

For all other equity positions, including net short positions, capital charges calculated under the PD/LGD approach may be no less than the capital charges that would be calculated using in (1) a 200% risk weight for publicly traded equity holdings and a 300% risk weight for all other equity holdings.

For more details we refer to Basel Committee on Banking Supervision (2004, p 73).

2.3 Market-Based Approach

Under the market-based approach, institutions are permitted to calculate the minimum capital requirements for their banking book equity holdings using one of the following methods: a simple risk weight method or an internal models method. The method that is actually used should be consistent with the amount and complexity of the institution's equity holdings and commensurate with the overall size and sophistication of the institution.

2.3.1 Simple Risk Weight Method

Under the simple risk weight method (SRWM), a 300% risk weight is to be applied in (1) to equity holdings that are publicly traded and a 400% risk weight is to be applied to all other equity holdings. A publicly traded equity holding is defined as any equity traded on a recognized security exchange.

Short cash positions and derivative instruments held in the banking book are permitted to offset long positions in the same individual stocks provided that these instruments have been explicitly designated as hedges of specific equity holdings and that they have remaining maturities of at least one year. Other short positions are to be treated as if they were long positions with the relevant risk weight applied to the absolute value of each position.

2.3.2 Internal Models Method

Banks may use, or may even be required by their supervisor to use, internal risk measurement models to calculate the risk-based capital requirement. According to the Basel Committee, banks may hold capital "equal to the potential loss on the institution's equity holdings as derived using internal value-at-risk models subject to the 99^{th} quantile, one-tailed confidence interval of the difference between quarterly returns and an appropriate risk-free rate computed over a long-term sample period"; see Basel Committee on Banking Supervision (2004, p 73). The capital charge would be incorporated into an institution's risk-based capital ratio through the calculation of risk-weighted equivalent assets. An example of an actuarial model, compliant with the philosophy of this Basel guideline in the area of Credit Risk, can be found in Dhaene *et al.* (2003).

3 Specifications of the "Internal Models Method"

3.1 Value at Risk

A risk measure summarizes the information contained in the distribution function of a r.v. (or risk) in one single real number. We conventionally assume that a negative value for the realisation of a risk means a loss whereas a positive value actually points to a gain. For an overview on the theory of risk measures, we refer to Dhaene *et al.* (2004a).

For a r.v. X, the p-quantile risk measure is defined as

$$Q_p(X) = \inf\{x | P[X \le x] \ge p\}, \qquad 0 < p < 1 . \tag{3}$$

Furthermore, the value at risk at a p-confidence level, denoted by $VaR_p(X)$, is now defined as

$$VaR_p(X) = Q_{1-p}(X) . \tag{4}$$

Consider a r.v. X which is normally distributed with mean μ and standard deviation σ. It is well-known that the quantiles of X are given by:

$$Q_p(X) = \mu + \phi^{-1}(p)\sigma . \tag{5}$$

We also have from (4) that

$$VaR_p(X) = \mu + \phi^{-1}(1-p)\sigma . \tag{6}$$

Let $P_0 = P > 0$ be the current price at time 0 of a particular investment portfolio, whereas P_t is its price at the end of the t^{th} period $(t = 1,2,\ldots,n)$. We define the log-return of the investment portfolio in the t^{th} period as

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right). \tag{7}$$

The cumulative log-return $R_{[k]}$ over k periods is now given by

$$R_{[k]} = \sum_{j=1}^{k} R_j . \tag{8}$$

If the subsequent log-returns R_t of the investment portfolio are independent and have identical normal distributions with mean μ and standard deviation σ then it follows that the value at risk over k periods $(k = 1,2,\ldots,n)$ of the investment portfolio is given by

$$VaR_p\left(P_{[k]}\right) = P\left(e^{\mu_{[k]} + \phi^{-1}(1-p)\sigma_{[k]}} - 1\right), \tag{9}$$

where $\mu_{[k]} = k\mu$ and $\sigma_{[k]} = \sigma\sqrt{k}$.

If the R_t 's are not normally distributed, but the deviations from normality are *"small"* enough, then we can approximate the non-normal distribution using the Cornish-Fisher expansion. In this case (9) becomes

$$VaR_p\left(P_{[k]}\right) = P\left(e^{\mu_{[k]} + \eta(1-p)\sigma_{[k]}} - 1\right), \tag{10}$$

with $\eta(p)$ given by

$$\begin{aligned}\eta(p) = {} & \phi^{-1}(p) + \frac{1}{6}\left(\left(\phi^{-1}(p)\right)^3 - 1\right)\gamma \\ & + \frac{1}{24}\left(\left(\phi^{-1}(p)\right)^3 - 3\phi^{-1}(p)\right)\kappa \\ & - \frac{1}{36}\left(2\left(\phi^{-1}(p)\right)^3 - 5\phi^{-1}(p)\right)\gamma^2\end{aligned} \tag{11}$$

In expression (11), γ and κ are the skewness and kurtosis of $R_{[k]}$ respectively. We refer to Dowd (2002, p 245) for more details on this approximation.

For the remainder of this paper we consider the length of a period to be equal to one quarter and $VaR_{99\%}\left(P_{[1]}\right)$ is then the value at risk at the one-quarter horizon with a 99% confidence level.

Under Basel's Value-at-Risk approach the regulatory capital CAP_{reg} of a portfolio and the corresponding risk weight RW are given by

$$CAP_{reg} = -VaR_{99\%}\left(P_{[1]}\right) \tag{12}$$

and

$$RW = 12.5\ CAP_{reg} \tag{13}$$

respectively.

$VaR_{99\%}\left(P_{[1]}\right)$ is the value at risk at the one-quarter horizon with a 99% confidence level.

3.2 Extreme Value Theory

Dowd (2002, p 271) points out that there are many problems in risk management that deal with extreme events – events that are unlikely to occur, but can be very costly when they do. These include large market falls, the outbreak of financial crises, etc. Consequently, risk management practitioners need to measure the risk associated with these extreme events.

However, estimation of VaR at extreme confidence levels is faced with a difficult problem: *as we have relatively few extreme observations on which to base our estimates, the standard error on the estimates of these VaR's can be significant and this uncertainty increases as our confidence level gets higher.*

To bypass these problems, practitioners resort to theory, or to be more precise, to statistical models which deal with extreme events. Researchers in these fields have developed a tailor-made approach – extreme value theory – that suits these sorts of problems. This approach focuses on the distinctiveness of extreme values and makes as much use as possible of what theory has to offer. The key to this approach is the extreme value theorem that tells us what the limiting distribution of extreme values should be.

Suppose we have a series of n independent subsequent random losses $\{X_k, k = 1,2,\ldots,n\}$. To begin with, we assume that the risks X_k are independently and identically distributed from some unknown distribution $F(x)$, and we want to estimate the maximal periodical loss we can encounter. Therefore, as a negative value of X_k represents a loss, we want to derive the d.f. of $\min\{X_k, k=1,2,\ldots,n\}$. Clearly, this poses a problem because we do not know what the d.f. $F(x)$ of the marginal risks X_k actually is.

However, under the above mentioned and other relatively mild assumptions, the Fisher-Tippett theorem, which can be seen as an equivalent of the Central Limit Theorem for extreme events, tells us that as n gets large, the d.f. of $\min\{X_k, k=1,2,\ldots,n\}$ converges to the following generalised extreme value distribution (GEV) $H_{\xi,\mu,\sigma}(x)$ given by:

$$H_{\xi,\mu,\sigma}(x)=\begin{cases}1-\exp\left[-\left(1-\xi\dfrac{(x+\mu)}{\sigma}\right)^{-1/\xi}\right] & \text{if } \xi\neq 0\\[2ex] 1-\exp\left[-\exp\left(\dfrac{(x+\mu)}{\sigma}\right)\right] & \text{if } \xi=0\end{cases}, \quad (14)$$

In (14) x satisfies the condition $1-\xi\dfrac{(x+\mu)}{\sigma}>0$.

Furthermore $H_{\xi,\mu,\sigma}(x)$ has three parameters which are:

- μ, the location parameter, which is a measure of central tendency,
- σ, the scale parameter, which is a measure of dispersion,
- ξ, the tail index, which gives an indication of the shape of the tail.

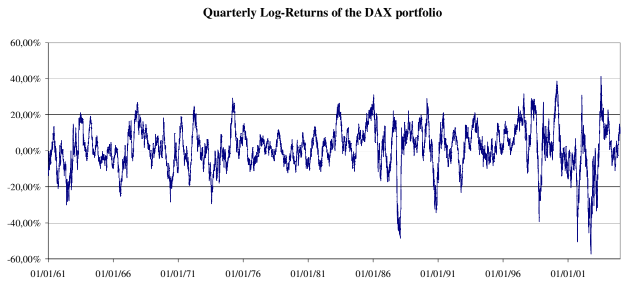
Depending to the level of the tail index, three different families can be identified for $H_{\xi,\mu,\sigma}(x)$:

- if $\xi > 0$, which means that $F(x)$ is fat-tailed, $H_{\xi,\mu,\sigma}(x)$ becomes the Fréchet distribution,
- if $\xi = 0$, the $H_{\xi,\mu,\sigma}(x)$ becomes the Gumbel distribution, corresponding to the case $F(x)$ has normal kurtosis,
- if $\xi < 0$, the $H_{\xi,\mu,\sigma}(x)$ becomes the Weibull distribution, corresponding to the case $F(x)$ has thinner than normal tails.

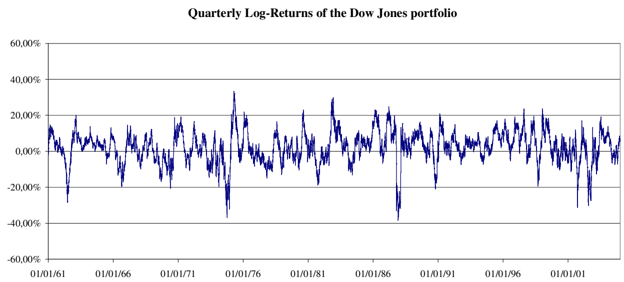
4 Regulatory Capital for Equity Portfolios

In this section we analyse the capital requirements for two equity portfolios, a "DAX Portfolio" which replicates the DAX index and a "Dow Jones Portfolio" which mimics the Dow Jones index. We assume that these portfolios are held by a German bank and a US bank respectively. Therefore there is no need to consider the currency risk.

Graph 1 and Graph 2 present the quarterly log-returns of the two portfolios from 1st January 1961 to 31st December 2004 calculated on a daily basis.



Graph 1: The graph presents, on a daily basis, the evolution of the quarterly log-returns of the DAX portfolio.

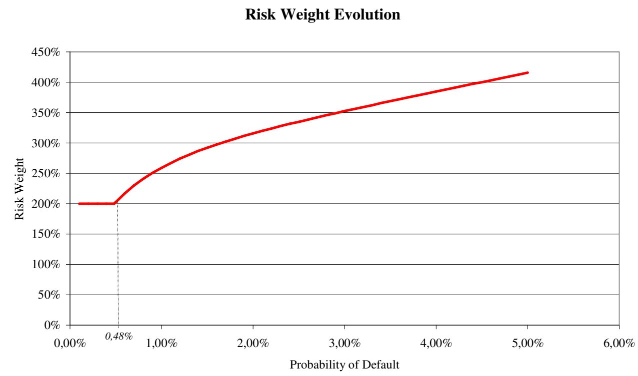


Graph 2: The graph presents, on a daily basis, the evolution of the quarterly log-returns of the Dow Jones portfolio.

4.1 Applying the PD/LGD Approach

We will assume that the PD's of the companies which compose the portfolio are the same, that the portfolio is managed in a way which does not allow the use of a RW of 100% and that the 1.5 scaling factor does not need to be applied.

From (2) we can then derive now how the portfolio risk weight varies with respect to the PD.



Graph 3: The graph presents the risk weight evolution regard to the PDs of the portfolio constituents assuming that their PDs are equal.

Notice that the lower bound of 200% for the risk weight applies whenever PD is smaller than 0.48%. As our portfolios only contain major traded companies, we can assume that their PDs are lower than 0.48%. Therefore the risk weight level which should eventually be applied to both portfolios is the lower bound, i.e. 200%. In other words, the regulatory capital for both portfolios is equal to 16% of the portfolio exposure.

4.2 Applying the Simple Risk Weight Method

As both portfolios are composed by exchange-traded equity exposures, the risk weight which should be used for the two portfolios is equal to 300%. Therefore the capital banks have to hold is equal to 24% of the portfolio exposure.

The simple risk weight method is quite simple to apply, but the other side of the coin is that it is too rough. Indeed, as we can notice from Graph 1 and Graph 2, the underlying risks of the two portfolios are not the same. Therefore we expect that the capital requirement varies for both portfolios, which is clearly not the case neither with this model nor with the PD/LGD one.

In the next sections, we analyse the capital requirement using VaR and EVT models.

4.3 Applying a Value at Risk Model

We will determine the value at risk assuming that the risks, i.e. the subsequent log-returns, are independent normal random variables. Table 2 presents the results we have obtained for the two portfolios we presented at the beginning of Section 4.

	DAX Portfolio	Dow Jones Portfolio
Log-Return Average	1.52%	2.16%
Log-Return Volatility	11.81%	8.80%
$Q_{1\%}\left(R_{[1]}\right)$	-25.96%	-18.30%
CAP_{reg}	22.86%	16.72%
RW	286%	209%

Table 2: The table details the VaR results obtained using a simple parametric VaR approach.

Table 2 shows that unlike the two previous approaches, using a simple parametric VaR the underlying risk characteristics of the portfolios are integrated in their *RW* levels.

The peaks in Graph 1 and 2 suggest that the log-returns are presumably not normally distributed and hence that the use of this simple parametric VaR could lead to the underestimation of the actual risks of both portfolios.

The table below presents the results we have obtained using the Cornish-Fisher expansion.

	DAX Portfolio	Dow Jones Portfolio
Log-Return Skewness	-0.55265	-0.51023
Log-Return Kurtosis	1.68342	1.33729
$\eta(0.01)$	-3.01133	-2.91619
$Q_{1\%}\left(R_{[1]}\right)$	-34.05%	-23.49%
CAP_{reg}	28.86%	20.93%
RW	361%	262%

Table 3: The table details the results obtained using the Cornish-Fisher approximation.

Generally speaking, VaR models determine the value of the loss we can encounter over the next period with a certain confidence level. Therefore, if banks hold a capital equal to the VaR, they should be saved from economic insolvency over this period of time. The optimality of VaR models has been discussed by Artzner *et al.* (1999) and also by Dhaene *et al.* (2004b). From a regulatory point of view, supervisors could go further in the financial protection of these institutions and say, for instance, that banks should protect themselves from the maximal loss they can encounter during one of the following periods. To determine this capital, traditional VaR models cannot be used. It is necessary to resort to extreme value theory.

However, it is not clear why banks should consider the paradigm of "maximal loss" rather than "actual loss" and Basel II does not provide any statement or support regarding the use of either philosophy.

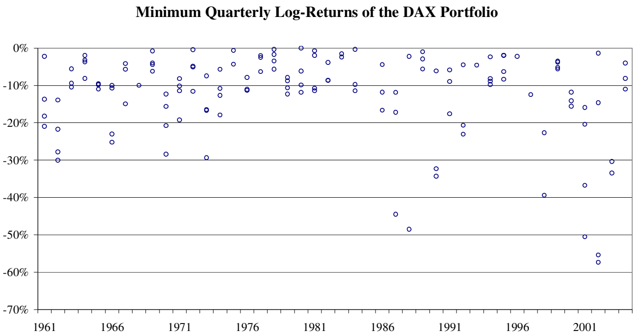
4.4 Extreme Value Theory

As extreme value theory deals with the maximum losses, we do not work with the total log-return series. From the original daily series, we select the minimum quarterly log-return observed each quarter. This technique is commonly known as the block maxima method, see Beirlant *et al.* (2004).

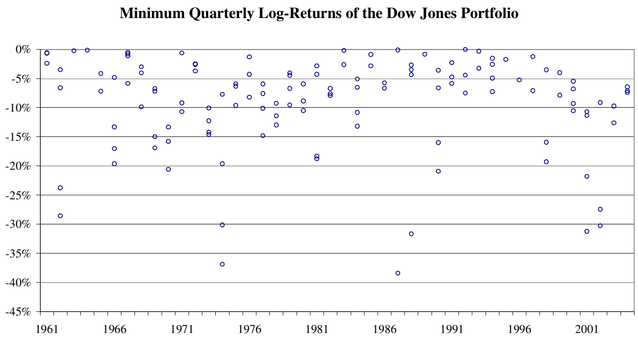
Assuming that each month contains 30 days, we define the minimum quarterly log-returns for a portfolio as:

$$R_t^{\min} = \min\left\{R_k, k \in \left[90\cdot(t-1)+1, 90\cdot(t-1)+90\right]\right\}, \quad \text{and } t = 1,2,3,\dots \tag{15}$$

Graph 4 and Graph 5 present the evolution of the minimum quarterly log-returns per year.



Graph 4: The graph presents the evolution of the minimum quarterly log-returns per year for the DAX portfolio.



Graph 5: The graph presents the evolution of the minimum quarterly log-returns per year for the DJ portfolio.

4.4.1 *Maximal Loss at 99% Confidence Level*

To estimate the unknown parameters of the generalised extreme value distributions, we use the maximum likelihood method.

In case $\xi \neq 0$, the log-likelihood function for the sample $R_1^{\min}$, …, $R_n^{\min}$ of i.i.d. GEV random variables is given by

$$\log L(\mu,\sigma,\xi) = -n\log\sigma - \sum_{t=1}^{n}\left(1-\xi\frac{R_t^{\min}+\mu}{\sigma}\right)^{-\frac{1}{\xi}} - \left(\frac{1}{\xi}+1\right)\sum_{t=1}^{n}\log\left(1-\xi\frac{R_t^{\min}+\mu}{\sigma}\right), \qquad (16)$$

provided $1+\xi\frac{R_t^{\min}-\mu}{\sigma} > 0$, t = 1, …, n. When ξ = 0, the log-likelihood function reduces to

$$\log L(\mu,\sigma,0) = -n\log\sigma + \sum_{t=1}^{n}\frac{R_t^{\min}+\mu}{\sigma} - \sum_{t=1}^{n}\exp\left(\frac{R_t^{\min}+\mu}{\sigma}\right). \qquad (17)$$

The ML estimator $(\hat{\mu},\hat{\sigma},\hat{\xi})$ for (μ,σ,ξ) is obtained by maximizing (16) and (17).

The regulatory capital of a portfolio is now given by:

$$CAP_{reg} = -P\left(e^{Q_{1-p}(R^{\min})}-1\right), \qquad (18)$$

where

$$\begin{cases} Q_{1-p}(R^{\min}) = -\hat{\mu} - \dfrac{\hat{\sigma}}{\hat{\xi}}\left(\left(\ln\left(\dfrac{1}{p}\right)\right)^{-\hat{\xi}}-1\right) & \text{if } \hat{\xi}\neq 0 \\ Q_{1-p}(R^{\min}) = -\hat{\mu} + \hat{\sigma}\ln\left(\ln\left(\dfrac{1}{p}\right)\right) & \text{if } \hat{\xi}=0 \end{cases}. \qquad (19)$$

The tables below presents the results we have obtained.

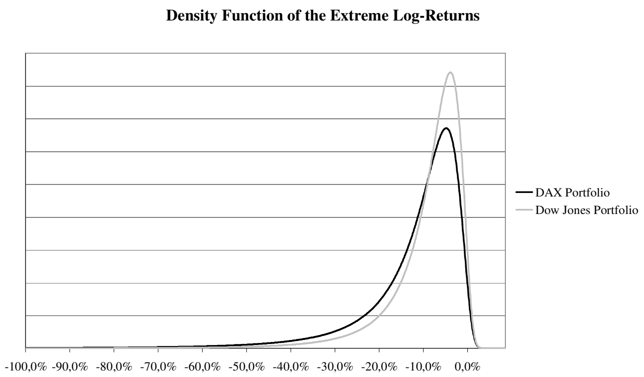
DAX Portfolio	
Parameter	Value
$\hat{\mu}$	0.06487
$\hat{\sigma}$	0.05775
$\hat{\xi}$	0.34275
$Q_{1\%}(R^{\min})$	-71.18%
CAP_{reg}	50.92%
RW	637%

Table 4: The table presents the results obtained for the DAX portfolio.

Dow Jones Portfolio	
Parameter	Value
$\hat{\mu}$	0.05006
$\hat{\sigma}$	0.04530
$\hat{\xi}$	0.27962
$Q_{1\%}(R^{\min})$	-47.44%
CAP_{reg}	37.77%
RW	472%

Table 5: The table presents the results obtained for the Dow Jones portfolio.

The graph below presents the density functions of the minimal values of the log-returns for both portfolios.



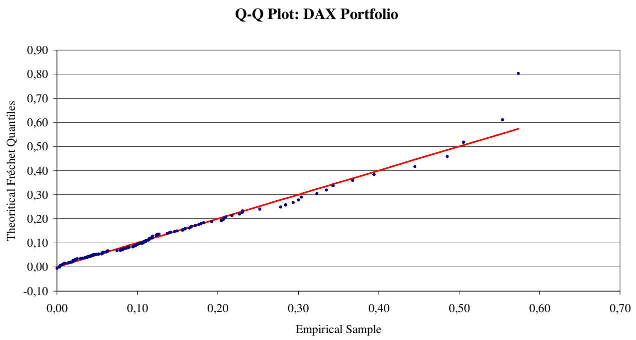
Graph 6: The graph presents the density functions of the minimum log-returns of the DAX and Dow Jones portfolios.

The next section presents a study to analyse the quality of the fittings.

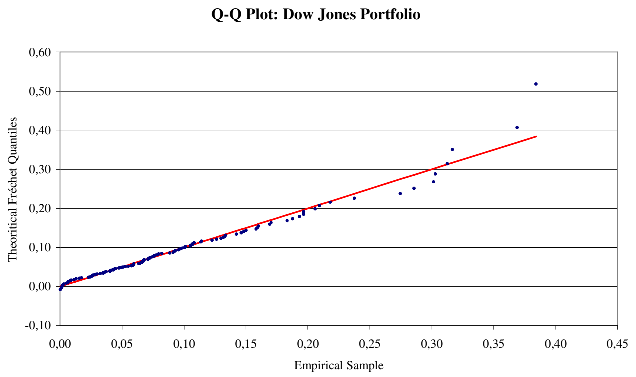
4.4.2 Goodness-of-Fit Tests

Q-Q plots compare empirical quantiles with the theoretical quantiles. They provide a visual assessment of the classical goodness-of-fit question, i.e. does a particular model provide a plausible fit to the distribution of the random variable at hand. Graph 7 and Graph 8 show the Q-Q plots we have obtained for the DAX and Dow Jones portfolios.

We point out that, in this paper, we have not presented the results obtained for both portfolios in the case $\xi = 0$ because the analyses of the Q-Q plots directly suggest that a Gumbel distribution does not fit adequately the empirical data.



Graph 7: Fréchet QQ-plot for the minimum log-returns of the DAX portfolio.



Graph 8: Fréchet QQ-plot for the minimum log-returns of the Dow Jones portfolio.

The visual check provided by these Q-Q plots is not sufficient to state that Fréchet distributions fit adequately the empirical data. Therefore we proceed to χ^2 goodness-of-fit tests. An attractive feature of this test is that it can be applied to any univariate distribution for which we can calculate the cumulative distribution function.

The χ^2-test is defined for the hypothesis:

$$\begin{cases} H_0: \text{ The data follow a Fréchet distribution,} \\ H_1: \text{ The data do not follow a Fréchet distribution.} \end{cases} \tag{20}$$

For the χ^2 goodness-of-fit computation, the data are divided into k bins and the test statistic is defined as

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}, \tag{21}$$

where O_i is the observed frequency for bin i and E_i is the expected frequency for bin i calculated by

$$E_i = N \cdot \left(H_{\xi,\mu,\sigma}(Y_u) - H_{\xi,\mu,\sigma}(Y_l) \right), \tag{22}$$

where $H_{\xi,\mu,\sigma}(x)$ is the Fréchet distribution, Y_u is the upper limit for class i, Y_u is the lower limit for class i, and N is the sample size. As far as the bin width is concerned, we use a class width equal to $0.3 \times s$, where s is the sample standard deviation.

The test statistic follows, approximately, a χ^2-distribution with $(k-c)$ degrees of freedom, where k is the number of non-empty bins and c is the number of estimated parameters + 1.

Therefore, the hypothesis that the data are from a population with the specified distribution is rejected if

$$\chi^2 > \chi^2_{(p,k-c)}, \tag{23}$$

where $\chi^2_{(p,k-c)}$ is the χ^2 percent point function with k-c degrees of freedom and a significance level of p.

The tables below presents the results we have obtained.

χ^2 Goodness-of-Fit Test for the DAX Portfolio	
Statistic	12.4763
Degree of Freedom	6
p-value	0.0521

Table 6: The table presents the key figures related to the χ^2 test performed for the DAX portfolio.

χ^2 Goodness-of-Fit Test for the Dow Jones Portfolio	
Statistic	13.5879
Degree of Freedom	7
p-value	0.0590

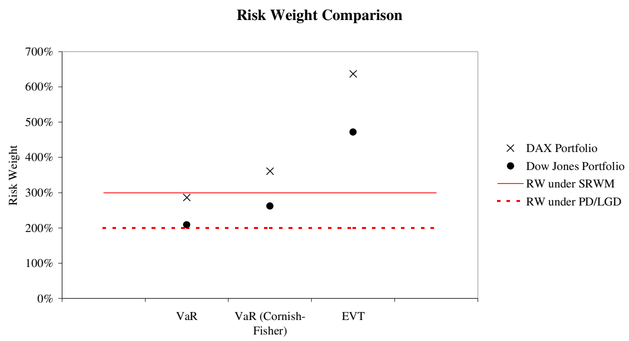
Table 7: The table presents the key figures related to the χ^2 test performed for the Dow Jones portfolio.

As p-values for both portfolios are greater than our subjective tolerance level of 5%, we conclude that we cannot reject the null hypothesis. This justifies the use of a Fréchet distribution.

5 Conclusion

In the first section of this document it was remarked that the ultimate goal of the Basel II Framework is to promote the adequate capitalisation of banks and to encourage improvements in risk management practices. Is this goal achieved?

The results obtained using the different Basel II methods are summarised in Graph 9.We believe that these results suggest that Basel II does not provide enough confidence regarding the adequate capitalisation of banks with respect to equity exposures. Indeed, the disparity of the RW levels obtained using the different Basel II-compliant methods inevitably makes the determination of the amount of the adequate regulatory capital difficult. It seems that these methods have not been jointly calibrated. Within this respect we notice that the use of the same confidence level for traditional VaR and extreme value models inevitably leads to a higher capital consumption for the latest ones.



Graph 9: The graph presents the level of RWs obtained using the different Basel II compliant methods.

Concerning the goal relative to the improvement in risk management practices, as Basel II allows the use of simple techniques (such as SRWM) which resulted in lower RW levels than the ones obtained with sophisticated techniques (such as EVT), there is no real incentive for banks to use these more sophisticated – and presumably more accurate – models.

Moreover, for such sophisticated techniques, we believe that appropriate confidence levels should be applied. A simple parametric VaR at a 99% confidence level cannot be compared with the maximal loss at a 99% confidence level determined using EVT.

At first sight, we could think that the use of internal models should allow for a better calculation of the regulatory capital a bank has to hold to protect itself from the risks it faces. However, if regulators do not clearly define the global framework of internal models (e.g. which VaR models must be used, what is the confidence level that should be used for each model, etc), they end up creating a situation which is the reverse to what they expected, i.e. better risk management practices.

We point out that these statements are made in the context of the calculation of regulatory capital for equity exposures (Pillar I). In general, equity exposures represent a tiny part of total bank exposures. Therefore, we are not arguing that Basel II leads to inappropriate global bank capitalisation. On the contrary Basel II is a major step forward vis-à-vis Basel I. However, it is key for regulators to adequately encourage the banks in the development of the most refined models.

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